PRIFYSGOL ABERYSTWYTH UNIVERSITY 1872

EffeCT

THE EXACT FACTORISATION OF MATRIX POLYNOMIALS

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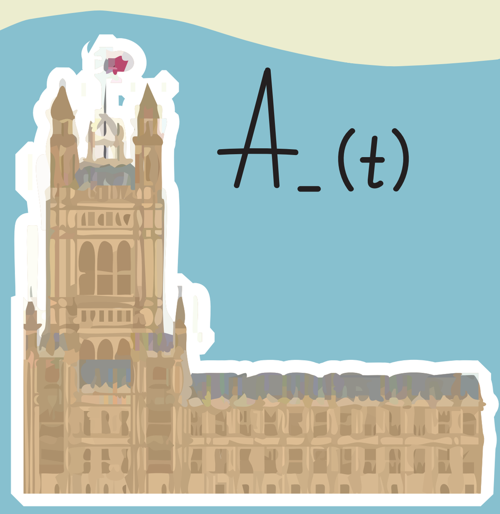
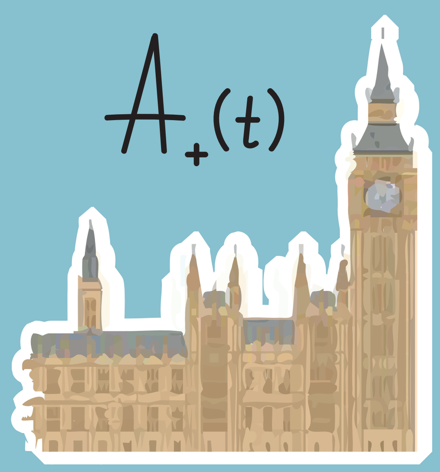
1. Wiener–Hopf Factorisation





$$A(t) = A_-(t)\, D(t)\, A_+(t)$$

2. Example

$$A(t) = \begin{pmatrix} 2t - t + \frac{1}{2} & -4t^2 + 2t - 1 \\ t & t^3 + t^2 + t + 4 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & -4 + \frac{2}{t} - \frac{1}{t^2} \\ \frac{35}{22} & \frac{3}{2} + \frac{3}{4t} + \frac{4}{t^2} \end{pmatrix}}_{A_-(t)} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & t^2 \end{pmatrix}}_{D(t)} \underbrace{\begin{pmatrix} \frac{44}{35} + \frac{121}{140}t + \frac{33}{70}t^2 & \frac{11}{70}t - \frac{11}{35}t^2 + \frac{22}{35}t^3 \\ -\frac{1}{2} & 1 \end{pmatrix}}_{A_+(t)}$$

Goal: to automate the factorisation of matrix functions and apply it in practice

3. Applications

Pure Mathematics

- boundary value problems (BVPs) in complex analysis (Riemann boundary value problem, Hilbert boundary value problem)
- a link between complex analysis and functional analysis: theory of linear operators in Banach spaces, Toeplitz operators, Wiener–Hopf operators and analytic theory of differential equations (Hilbert's 21st problem)

Applied Mathematics

- systems of singular integral equations with the Cauchy kernel, a kernel depending on the difference of the arguments, and systems of an infinite number of equations, etc.
- BVPs for systems of differential equations in partial derivatives with mixed boundary conditions

Mechanics

- of elastic and viscoelastic solids with defects
- waves in solids and structures with defects

Mechanical and Structural Engineering, Geomechanics, Fracture Mechanics, Maritime Engineering, etc.

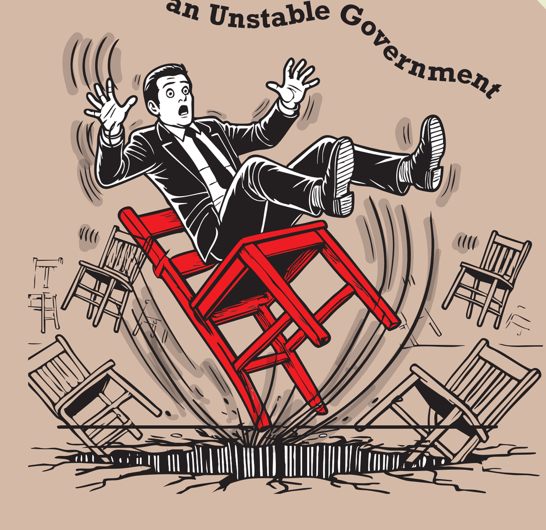
Physics

- solution of nonlinear equations of mathematical physics by the method of the inverse scattering problem
- theory of solitons
- diffraction of electromagnetic and acoustic waves
- problems of geophysics

Financial Mathematics

- pricing barrier options
- risk management
- time series analysis

4. Obstacles



Obstacle 1: absence of the explicit formulae for constructing factorisation and calculating partial indices

Obstacle 2: the factorisation problem is not always stable

5. Solutions

Pre-election polls

Solution 1: concentrate on the class of matrix polynomials

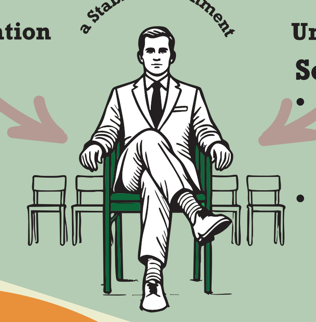
Company I

Company II

Scenario 1: polling results match (XYZ and XYZ)

Scenario 2: polling results differ (XYZ and YZX)

Stable factorisation



Unstable factorisation

Solution 2:

- solve the problem exactly
- prove the existence criterion for the exact solution to the factorisation problem for matrix polynomials

a NEW ERA in FACTORISATION: the COMPUTATION REVOLUTION !

6. Our Package: the ExactMPF

>_ a p x p matrix polynomial

The package checks: is it possible to find the exact factorisation for it?

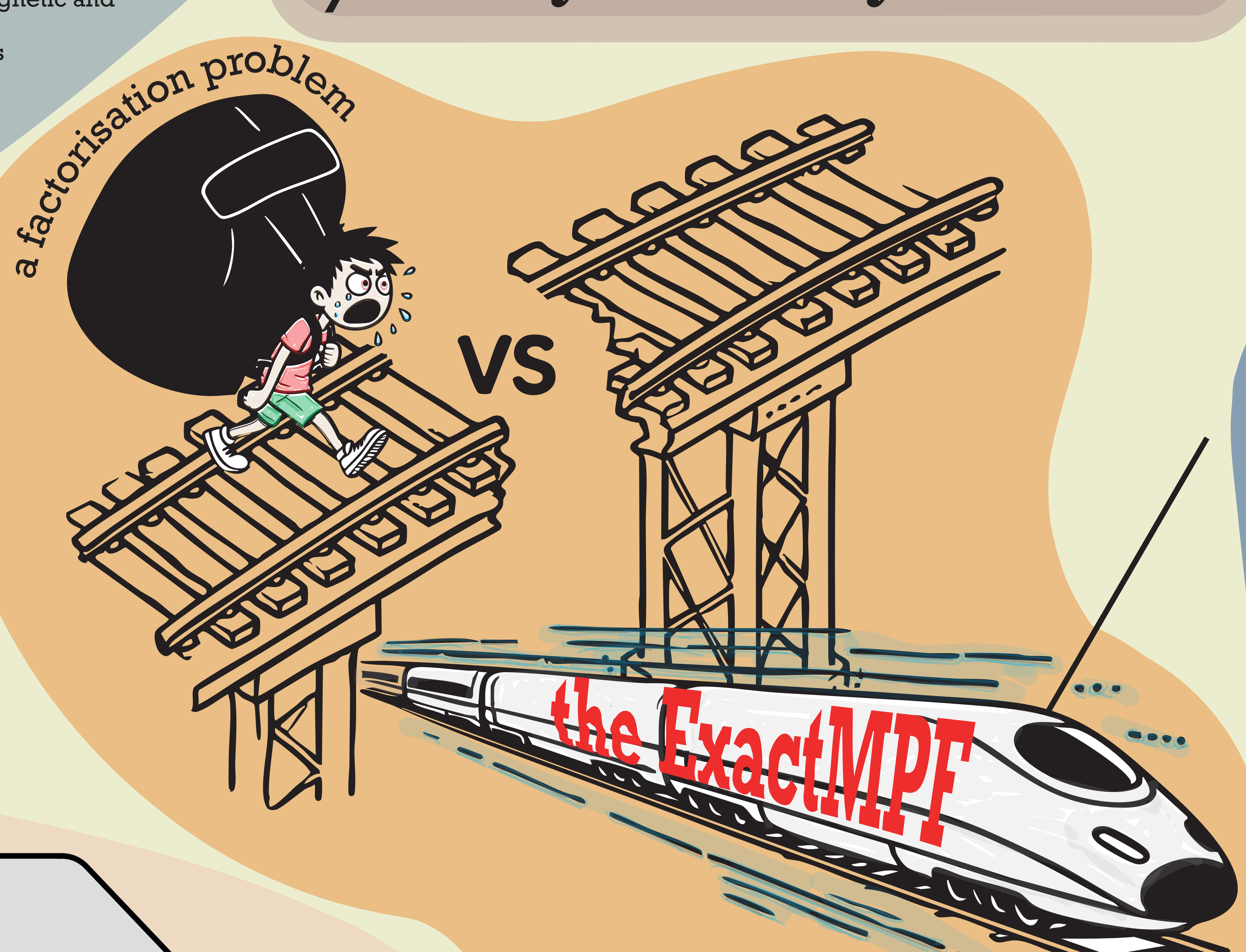
YES

- the right and left factorisations
- the partial indices

NO

the exact factorisation is not possible

7. Why should you care?



Our package:

- **the first and only one package** in which the factorisation can be performed
- **the process** of constructing the factorisation is **fully automated**
- **the right** and **left factorisations** are found **simultaneously**

The Wiener–Hopf factorisation method:

- **is a step towards solving a 60-year-old problem** in unstable matrix factorisation
- **unlocks new applications**, among others, in quantum information science

8. The ExactMPF: Success Cases

- the numerical solution of the discrete Schrödinger equation (**Fig.1** and **Fig.2**)
- a stable factorisation of strictly nonsingular 2 × 2 matrix functions (**Fig. 3** and **Fig.4**)

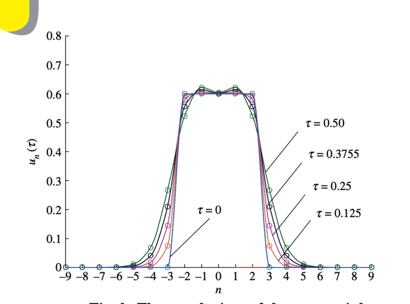


Fig 1. The evolution of the potential using the ExactMPF package

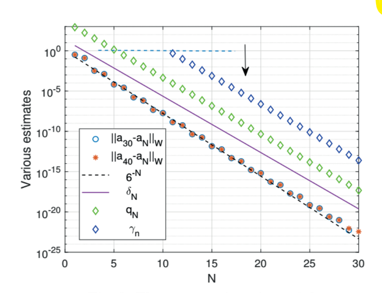


Fig 3. The approximation of the factors

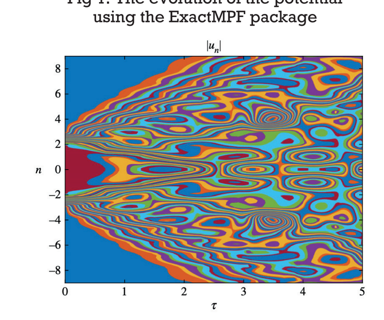


Fig 2. Stable reconstruction of wave dynamics with the ExactMPF package

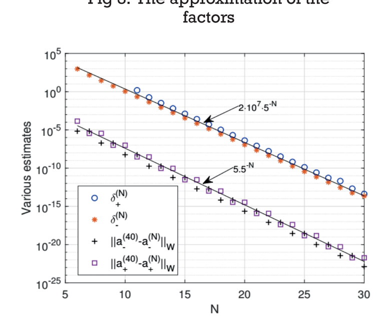


Fig 4. The exact factorisation of the given matrix

- factorisation of piecewise constant matrix functions

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