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The Wiener-Hopf Technique and its Applications

Techneg Wiener-Hopf a'i Chymwysiadau

ABSTRACTS

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An Application of the Wiener-Hopf Technique to the Pricing of Options in Mathematical Finance

I. D. Abrahams

University of Manchester

This talk will offer a very brief introduction to the pricing of financial options. Although these financial instruments have been criticised in recent years because of their use by speculators, they are still widely employed in the business world for hedging.

We shall show that certain types of options can be modelled within the Black-Scholes framework in terms of integral relations, which, via z-transforms, can be reduced to a Wiener-Hopf equation. We shall study this equation and indicate how the formal solution can be evaluated via Pade approximants.

As time allows we will relate the work to other diverse physical models which use z-transforms and/or Pade approximant methods in Wiener-Hopf theory.

Propagation of Slepyan's Crack in a Non-Uniform Elastic Lattice

M. J. Nieves[1], A. B. Movchan[2], I. S. Jones[1], G. S. Mishuris[3]

[1]*Liverpool John Moores University*, [2]*University of Liverpool*, [3]*Aberystwyth University*

We model the propagation of a Mode I semi-infinite crack through an infinite discrete triangular lattice, with bonds having a contrast in stiffness in the principal lattice directions. This non-uniformity within the lattice can be interpreted, for example, as the effect of thermal pre-stress of a constrained lattice whose ligaments have different coeficients of thermal expansion. The propagation of the crack can be accompanied by an external load generating feeding waves which bring energy to the crack front bonds and cause their disintegration one by one. In turn this produces dissipative waves which carry energy away from the crack vertex. Using the approach of Slepyan [1], an equation of the Wiener-Hopf type is derived and solved along the crack face. The kernel function of this equation is linked to the corresponding Green's kernel for the problem. Analysis of this kernel leads to explicit wave dispersion dependencies for the crack within the lattice. The crack stability is analysed via the evaluation of the energy release rate for different contrasts in stiffness of the bonds. Using this model, similar to Slepyan et al. [2], we present numerical illustrations showing that the average crack speed for a Mode I crack within a thin strip of triangular lattice can be predicted from the dispersion relations for a given frequency of the remote source.

References

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Failure Waves in Periodically Supported Elastic Beams

MICHELE BRUN[1,2], ALEXANDER B. MOVCHAN[3], LEONID I. SLEPYAN[4]

[1] *Cagliari University*, [2] *Cittadella Universitaria*, [3] *University of Liverpool*, [4] *Tel Aviv University*

We consider damage propagation in an elongated periodically supported elastic structure like a bridge. The failure is modeled as a drop of the stiffness of the supports when a critical value of the stress is reached. We consider three different models: (a) the continuous inertial beam on a distributed elastic foundation of the Winkler-type, (b) the discrete one composed by a set of masses rested on elastic supports and connected by massless beams and (c) the discrete-continuous dynamic beam on the set of discrete elastic supports. Conditions for the failure wave to exist, to propagate uniformly or to accelerate are established and the results of the three models are detailed and compared. For the beam on distributed elastic foundation three regimes, subsonic, intersonic and supersonic, are established and it has been found that uniform propagation is possible only in the intersonic regime. Condition of propagation has been computed for the discrete model and the dynamic beam on discrete supports implementing the Wiener-Hopf technique. In the latter case, a convergent integral with highly oscillating integrand has been regularised introducing a small dissipation proportional to the strain rate, that is a small viscosity. It is shown that when the distance a between the discrete supports, namely the span length of the bridge, is sufficiently small the displacement in the separation point for the discrete and discrete-continuous models converges to the value of the continuous model with distributed supports. Surprisingly, the limiting velocity for uniform propagation is slightly in uenced by the distance a in a large interval of a.

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Wiener-Hopf and Wiener-Hopf-Hankel Operators with Piecewise-Almost Periodic Symbols on Weighted Lebesgue Spaces

LUIS CASTRO

Aveiro University

We consider Wiener-Hopf, Wiener-Hopf plus Hankel, and Wiener-Hopf minus Hankel operators on weighted Lebesgue spaces and having piecewise almost periodic Fourier symbols. The main results concern conditions to ensure the Fredholm property and the lateral invertibility of these operators. In addition, under the Fredholm property, conclusions about the Fredholm index of those operators are also discussed. The talk is based on a joint work with Anabela Silva.

Asymptotic Analysis of Interface Crack Problems for Metallic-Piezoelectric Composite Structures

Otar Chkadua[1], David Natroshvili[2]

[1] *Tbilisi State University*, [2] *Georgian Technical University*

We investigate three–dimensional interface crack problems for metallic-piezoelectric composite bodies with regard to thermal effects. We give a mathematical formulation of the physical problems when the metallic and piezoelectric bodies are bonded along some proper parts of their boundaries where interface cracks occur. By the potential method the interface crack problems are reduced to equivalent strongly elliptic systems of pseudodifferential equations on manifolds with boundary. We study the solvability of these systems in appropriate function spaces and prove uniqueness and existence theorems for the original interface crack problems. We analyse the regularity and asymptotic properties of the corresponding thermo-mechanical and electric fields near the crack edges and near the curves where the different boundary conditions collide. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators. For some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and show that they essentially depend on the material parameters. The questions related to the so called oscillating singularities are treated in detail as well.

Stroh Formalism for Interface Crack Problems in Anisotropic Materials: An Alternative Approach for Avoiding Wiener-Hopf Technique

L. Morini[1], E.Radi[1], A.B.Movchan[2], N.V.Movchan[2], A.Piccolroaz[3,4], G.S.Mishuris[3]

[1] *Universitá di Modena e Reggio Emilia*, [2] *University of Liverpool*, [3] *Aberystwyth University*, [4] *Trento University*

We consider quasi-static problem for a semi-infinite crack lying at the interface between two dissimilar anisotropic elastic materials and loaded by a general asymmetrical system of forces acting on the crack faces. The weight functions, both symmetric and skew-symmetric, can be identified as a non-trivial singular solutions of the homogeneous boundary value problem for the solid with a crack. Conventionally, the problem is reduced to solving a challenging matrix Wiener-Hopf functional equation. The Stroh matrix representation of displacements and tractions, combined with a Riemann-Hilbert formulation, provides an alternative approach which is used to obtain an algebraic eigenvalue problem that is solved in a closed form. Symmetric and skew-symmetric weight functions matrices for both plane strain and antiplane strain crack problems are derived by means of this new approach. The weight functions are then used together with Betti integral formula in the computation of the stress intensity factors corresponding to an example of asymmetric load acting on the crack faces. The derived weight functions are used to evaluate 2D integral identities in terms of singular integral equations matching tractions and the resulting crack opening along the crack surfaces. The derived compact formulation can be used to solve problems in linear inisotropic elastic fracture mechanics as well as potential applications in multiphysics modelling, where the elastic problem is coupled with other concurrent physical phenomena.

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Water-Wave Reflection by Submerged Thin Horizontal Plates

Malte Peter
Augsburg University

On the basis of linear theory, the Wiener-Hopf technique is used to derive an explicit expression for the reflection coeffcient when a plane wave is obliquely incident upon a submerged semi-infinite porous plate in water of finite depth. Having used the Cauchy Integral Method in the factorisation, the expression does not rely on knowledge of any of the complex-valued eigenvalues or corresponding vertical eigenfunctions in the region occupied by the plate. It is shown that the Residue Calculus technique yields the same result as the Wiener-Hopf method for this problem and this is also used to derive an analytical expression for the solution of the corresponding finite-plate problem. Extensions to submerged elastic plates are discussed as well.

Fracture in Microstructured Materials

E.Radi[1], A.Piccolroaz[2,3], G.S.Mishuris[2]
[1] *Universitá di Modena e Reggio Emilia*, [2] *Aberystwyth University*, [3] *Trento University*

This paper is concerned with the problem of a semi-infinite crack steadily propagating in an elastic solid with microstructures subject to antiplane loading applied on the crack surfaces. The loading is moving with the same constant velocity as that of the crack tip. We assume subsonic regime, that is the crack velocity is smaller than the shear wave velocity. The material behaviour is described by the indeterminate theory of couple stress elasticity developed by Koiter. This constitutive model includes the characteristic lengths in bending and torsion and thus it is able to account for the underlying microstructure of the material as well as for the strong size effects arising at small scales and observed when the representative scale of the deformation field becomes comparable with the length scale of the microstructure, such as the grain size in a polycrystalline or granular aggregate. The present analysis confirms and extends earlier results on the static case by including the effects of crack velocity and rotational inertia. By adopting the criterion of maximum total shear stress, we discuss the effects of microstructural parameters on the stability of crack propagation.

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Localized Boundary-Domain Integral Equations for Acoustic Scattering by an Inhomogeneous Anisotropic Obstacle

Otar Chkadua[1], Sergey E. Mikhailov[2], David Natroshvili[3]

[1] *Tbilisi State University*, [2] *Brunel University*, [3] *Georgian Technical University*

We consider the time-harmonic acoustic wave scattering by a bounded *anisotropic inhomogeneity* embedded in an unbounded *anisotropic* homogeneous medium. The material parameters and the refractive index are assumed to be discontinuous across the interface between the inhomogeneous interior and homogeneous exterior regions. The corresponding mathematical problem is formulated as a transmission problem for a second order elliptic partial differential equation of Helmholtz type with discontinuous variable coefficients. We show that the transmission problem with the help of *localized potentials* can be reformulated as a *localized boundary-domain integral equations* (LBDIE) system and prove that the corresponding *localized boundary-domain integral operator* (LBDIO) is invertible.

First we establish the equivalence between the original transmission problem and the corresponding LBDIE system which plays a crucial role in our analysis. Afterwards, we establish that the localized boundary domain integral operator obtained belongs to the Boutet de Monvel algebra of pseudodifferential operators. And finally, applying the Vishik-Eskin theory based on the factorization method (the Wiener-Hopf method) we investigate Fredholm properties of the LBDIO and prove its invertibility in appropriate function spaces. This invertibility property implies then existence and uniqueness results for the LBDIE system and the corresponding original transmission problem.

Beside a pure mathematical interest these results can be applied in constructing and analysis of numerical methods for solution of the LBDIEs and thus the scattering problems in inhomogeneous anisotropic media.

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Mixed Type Boundary Value Problems for Polymetaharmonic Equations

George Chkadua

Tbilisi State University

We consider three dimensional mixed boundary value problems for the polymetaharmonic equation with real wave numbers k_1 and k_2,

$$\left(\Delta + k_1^2\right)\left(\Delta + k_2^2\right) u = 0 \qquad \text{in} \quad \Omega^-,$$

where Ω^- is an unbounded domain in $\mathbb{R}^3$ with compact boundary $S = \partial\Omega^- \in C^\infty$; $\Omega^+ = \mathbb{R}^3 \setminus \overline{\Omega^-}$.

Let the boundary $S = \partial\Omega^+ = \partial\Omega^-$ be divided into two disjoint parts S_1 and S_2, $S = \overline{S_1} \cup \overline{S_2}$, $S_1 \cap S_2 = \varnothing$ and $l_m := \partial S_1 = \partial S_2 \in C^\infty$. Further, let

$$H^1_{loc}(\Omega^-, \Delta) := \{u : u \in H^1_{loc}(\Omega^-),\ \Delta u \in H^1_{loc}(\Omega^-)\}.$$

We consider the following mixed boundary value problems (cf. [1], [2]).

Riquier-type mixed problem (MR): Find a radiating solution $u \in H^1_{loc}(\Omega^-, \Delta)$ of the polymetaharmonic equation satisfying the mixed boundary conditions

$$\begin{aligned} &\{u\}^-_{S_1} = f_0, \quad \{\Delta u\}^-_{S_1} = f_1 && \text{on} \quad S_1, \\ &\{\partial_n u\}^-_{S_2} = g_0, \quad \{\partial_n \Delta u\}^-_{S_2} = g_1 && \text{on} \quad S_2, \end{aligned}$$

where $f_0, f_1 \in H^{1/2}(S_1)$, $g_0, g_1 \in H^{-1/2}(S_2)$.

Sobolev-type mixed problem (MS): Find a radiating solution $u \in H^2_{loc}(\Omega^-)$ of the polymetaharmonic equation satisfying the mixed boundary conditions

$$\begin{aligned} &\{\mathbf{B}_0 u\}^-_{S_1} = f_0, \quad \{\mathbf{B}_1 u\}^-_{S_1} = f_1 && \text{on} \quad S_1, \\ &\{\mathbf{B}_2 u\}^-_{S_2} = g_0, \quad \{\mathbf{B}_3 u\}^-_{S_2} = g_1 && \text{on} \quad S_2, \end{aligned}$$

where $\mathbf{B}_0 := I$, $\mathbf{B}_1 := \partial_n$, $\mathbf{B}_2 := -\partial_n^2$, $\mathbf{B}_3 := 2\partial_n\Delta - \partial_n^3 + (k_1^2 + k_2^2)\partial_n$, and $f_0 \in H^{3/2}(S_1)$, $f_1 \in H^{1/2}(S_1)$, $g_0 \in H^{-1/2}(S_2)$, $g_1 \in H^{-3/2}(S_2)$.

We investigate these problems by means of the potential method and the theory of pseudodifferential equations on a manifold with boundary based on the Wiener-Hopf (factorization) method. We prove the existence and uniqueness of solutions in Sobolev-Slobodetski spaces. The solutions of the mixed BVP (**MS**) have higher smoothness (H^2-smoothness) than the solutions of the classical Riquier type mixed BVP (**MR**) which belong to H^1 in general.

Mixed problems belong to the class of problems whose solutions along with their derivatives have singularities near the *exceptional curve* l_m (the curve where the different boundary conditions collide) even for infinitely smooth boundary data. We analyse the asymptotic behavior of solutions near the exceptional curve and derive that solutions of the Riquier type mixed problem (**MR**) have $C^{1/2}$-smoothness, while solutions of the mixed problem (**MS**) have higher $C^{3/2}$-smoothness at the exceptional curve l_m.

References

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Local Principle and SIO on Nilpotent Lie Groups

Vladimir Kisil

University of Leeds

We describe connections between the localization technique introduced by I.B. Simonenko and operator covariant transform produced by nilpotent Lie groups.

Constructive Matrix Functions Factorisation

Sergei Rogosin

Belarusian State University

A survey of recent results on matrix-functions factorization will be pre- sented. Special attention will be paid to piece-wise smooth matrix-functions.

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Effective Approximate Solution for a Scalar Wiener-Hopf Problem

Anastasia Kisil

Cambridge University

A novel method of approximating the scalar Wiener-Hopf equation; and hence constructing an approximate solution will be presented. The advantages of this method over the existing methods are its speed, reliability and explicit bounds on the error. The degrees of the polynomials in the rational approximation are smaller then in other methods and produce much smaller error.

Fracture and waves propagation in bimaterial lattice structure

Andrea Piccolroaz[1,2], Gennady Mishuris[1]

[1]*Aberystwyth University*, [2]*Trento University*

We consider a rectangular bimaterial lattice structure with interface links having different properties than those of the glued structures. The fault propagates along the interface generating waves. We construct dispersion diagrams analysing possible waves propagating in such structure. Then we consider the steady-state fault propagation on the basis of the approach of Prof. Slepyan. We show that the corresponding matrix Wiener-Hopf equation can be decoupled and effectively factorised.

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