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ASYMPTOTIC MODELLING IN ARTICULAR CONTACT MECHANICS

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Introduction

Articular cartilage is the off-white firm, visco-elastic tissue covering the end of each bone at the joints. Cartilage has no known function other than maintaining mechanical competence of joints, allowing bones to move against one another without friction. But there is no need to underline its significance to health of a human body, since almost all the load transmitted by a human joint goes through the articular cartilage, and it prevents biomechanical damage caused by severe loading.

Osteoarthritis (OA) is a degenerative disease which involves deterioration of articular cartilage and affection of the underlying bone. Despite the large amount of new information gleaned from in vitro and in vivo studies on OA, there remain challenges to understand how to modify the processes involved in OA pathogenesis[1]. Today, there is no effective medical treatment of the OA degradation of articular cartilage[2], and the only feasible options are to resurface or totally replace the joint surgically.

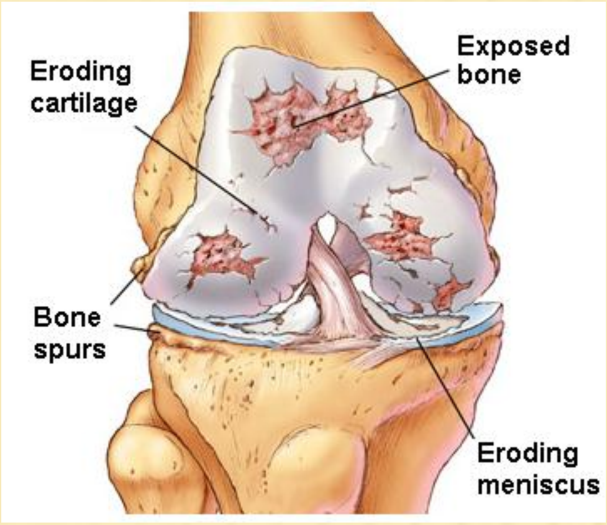

Fig. 1. *OA Knee joint*

The most common target joint affected by OA is the knee joint. The forces applied to the knee during normal activities are significant. For a patient standing on a level surface, the load is about half of body weight. Experimental investigations have shown that joint contact forces of up to 300% body weight can occur even during normal walking, and rise to 550% during the push-off phase of running. Though the degeneration process at the OA knee joint is believed to result from a combination of mechanical loading and biological weakening of the cartilage matrix, considerable epidemiological evidence supports the concept that mechanical conditions producing increased load transfer across the joint and the altered contact pressure patterns can accelerate the initiation and progress of OA.

Methods

At present, many mathematical models developed for describing articular contact mechanics become so analytically complex that only numerical solutions by computer seem to be feasible. This comes in apparent contradiction with the necessity to have a clear understanding of the underlying principles of OA, the causes of variation in articular cartilage properties, the effects of gradual and subtle changes in cartilage at the onset of OA and its progression upon the evolution of the contact pattern.

Asymptotic Modelling (AM) is an analytical mathematical technique of analyzing and simplifying mathematical models[3]. AM being applied to the existing mathematical models of biomechanical contact would aid in understanding of articular joint mechanics and etiology of the OA process. The AM approach that is proposed for getting an insight into articular cartilage contact is an analytical approach in mathematical modelling which combines generality of the problem statement with a sufficient accuracy of asymptotic solutions obtained.

Results

Ateshian *et al.* (1994) obtained asymptotic solution for the axisymmetric articular contact problem. Wu *et al.* (1996) extended this solution to a more general model combining with the assumption of the kinetic relationship from Hertz's contact mechanics with the joint contact model for two biphasic cartilage. A general solution of axisymmetric articular contact problem for two biphasic cartilage layers was obtained by Argatov (2010). Mishuris and Argatov (2009) and Argatov and Mishuris (2010) extended the analysis of Wu *et al.* (1996) by formulating the refined kinetic relationship accounting for the tangential displacements at the contact region.

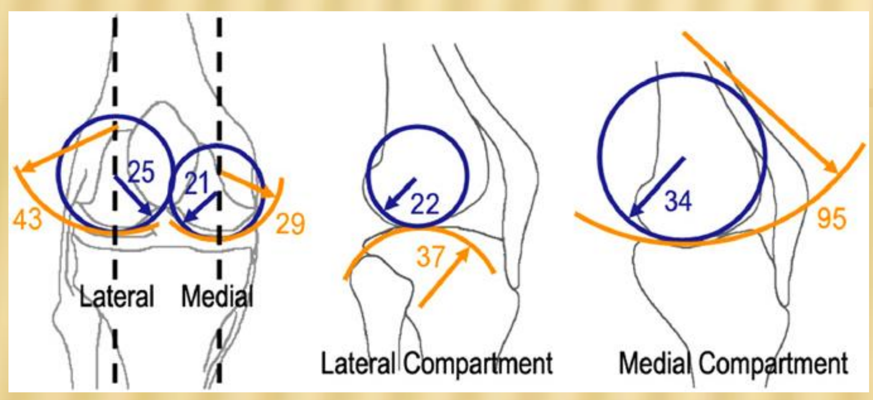

Fig. 2. *Elliptical contact configuration* [6]

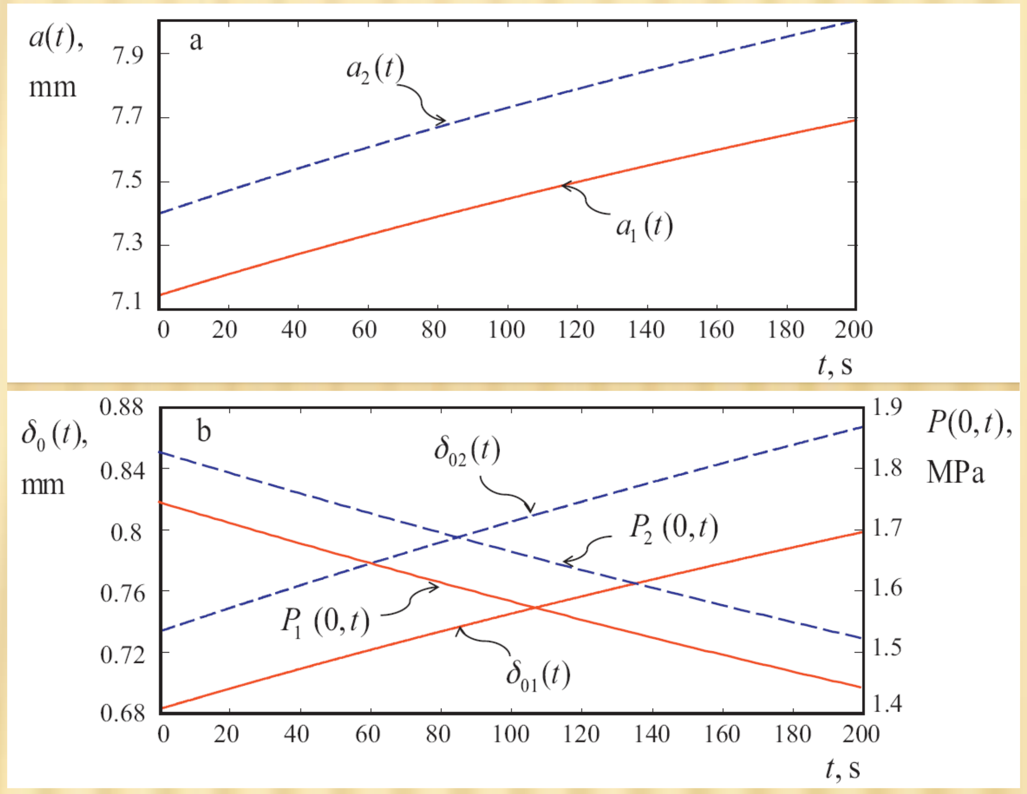

Fig. 3. *Prediction of contact parameters* [9]

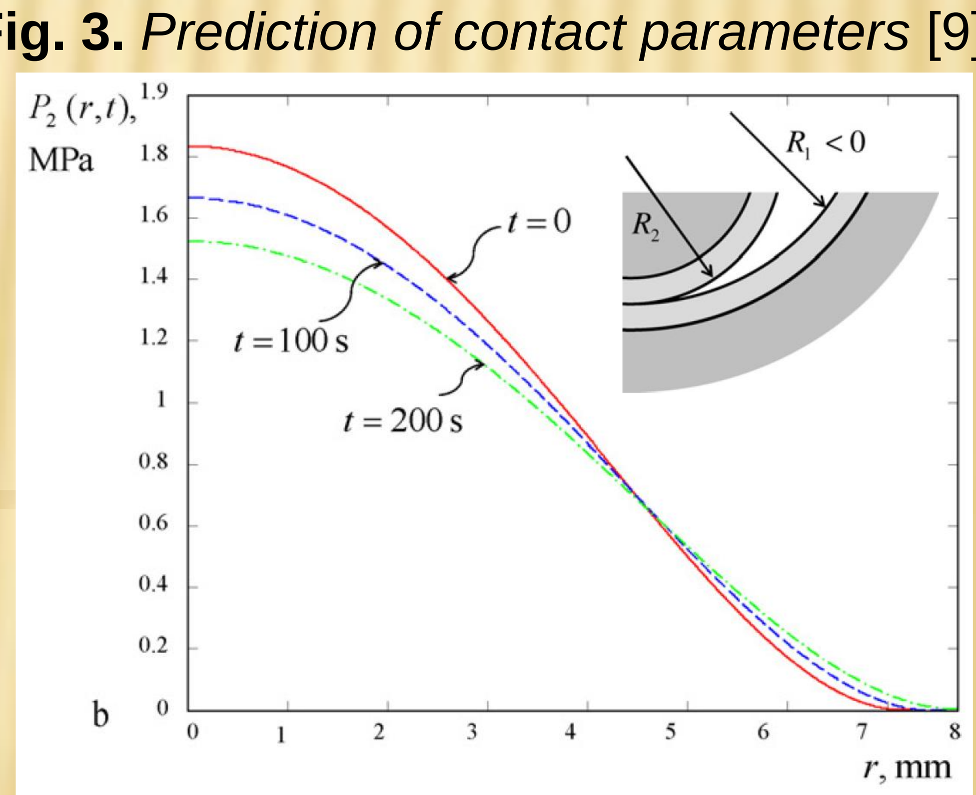

Fig. 4. *Prediction of contact pressures* [9]

It should be emphasized that axisymmetric assumption is a limitation of the mentioned above analytical solutions. The axisymmetric model of articular contact mechanics (Ateshian *et al.*, 1994; Wu *et al.*, 1996) was generalized for the three-dimensional case by Argatov and Mishuris (2010) and the exact closed-form solution of the articular contact problem for biphasic cartilage layers shaped like elliptic paraboloids was obtained.

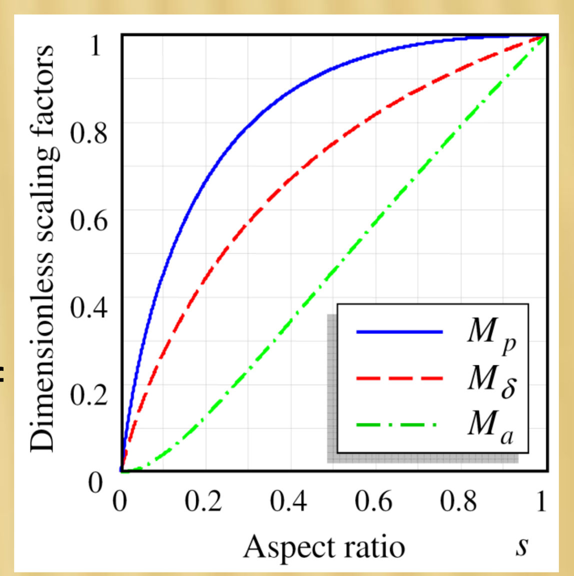

Fig. 5. *Elliptical contact parameters* [10]

Recently, Argatov and Mishuris (2011) developed asymptotic methodology for modelling tibio-femoral contact which is capable of accurately describing the mechanical behavior of articular cartilage layers near the compaction point. The exact analytical solution to the articular contact problem is obtained in a closed form. The new contact model for tibio-femoral contact incorporates the previously developed (Wu *et al.*, 1996) axisymmetric asymptotic model.

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