Mathematical Modelling of Anisotropic Bimaterials With Imperfect Interfaces

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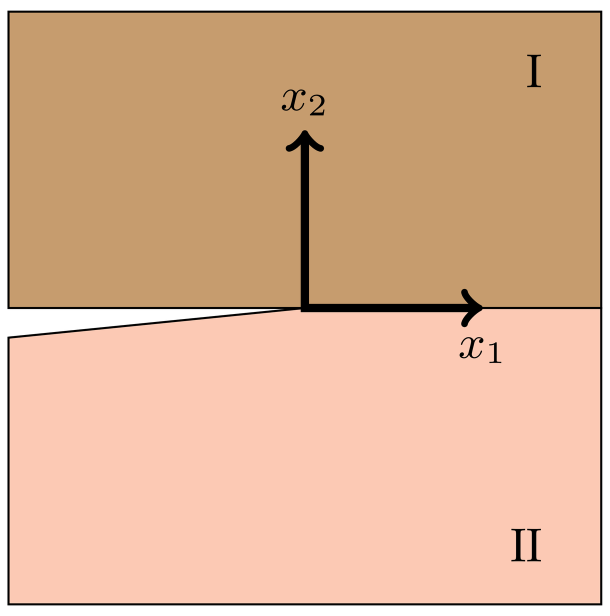
Introduction

Crack propagation along interfaces in elastic bi-materials is an area of great interest in applied mathematics. The geometry of such problems is as follows:



The following properties of the system have a great effect on the nature of the desired solutions of the problem:

- Elastic properties of the materials (i.e. isotropic, anisotropic etc.),
- Continuity of displacements and traction over the bonded portion of the interface between the materials,
- Whether the interfacial crack considered is stationary or moving,
- Physical loading applied on the crack faces.

Different combinations of these properties have been considered, including the study of a moving crack in an anisotropic bimaterial with a perfect interface in [1] and [2].

Problem formulation

The problem considered is that of an interfacial crack along an imperfect interface in an anisotropic bimaterial. An imperfect interface between materials exhibits continuity of traction, $\mathbf{t}$, but discontinuity of displacement, $\mathbf{u}$.

Mathematically, the boundary conditions are given as:

$$\mathbf{t}(x_1, 0^{\pm}) = \mathbf{p}^{\pm}(x_1), \quad \text{for } x_1 < 0,$$

$$\mathbf{t}(x_1, 0^{+}) = \mathbf{t}(x_1, 0^{-}), \quad \text{for } x_1 > 0,$$

$$[\![\mathbf{u}]\!](x_1) = \begin{pmatrix} K_{11} & K_{12} & 0 \\ K_{12} & K_{22} & 0 \\ 0 & 0 & \kappa \end{pmatrix} \mathbf{t}(x_1, 0^{+}), \text{ for } x_1 > 0,$$

where the function $\mathbf{p}(x_1)$ is known. Constants K_{11}, K_{12}, K_{22} and κ describe the extent of interface imperfection. $[\![\mathbf{u}]\!]$ is the displacement jump.

Solutions are sought for the tractions ahead of the crack tip and the jump in displacement over the crack. In order to find these the following mathematical methods are used:

- Weight Functions,
- Betti Identity,
- Singular Integral Equations.

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Integral equations for out of plane shear (Mode-III)

Weight functions are functions whose weighted integrals are used to derive key values (such as stress intensity factors) to investigate the behaviour of a propagating crack. The work here uses a **novel method** where known weight functions can be used to find displacements and stresses for an anisotropic bimaterial with an imperfect interface. This **negates the need to find new weight functions** as the ones required can be found in [1] and [2].

Using this approach, the following integral equations are found in [3]:

$$-\frac{\mathcal{H}_{33}}{\pi\kappa}\mathcal{T}^{(s)}_{\mathcal{H}_{33}}[\![u]\!]^{(-)} - \frac{1}{\kappa}[\![u]\!]^{(-)} - \frac{1}{\pi\kappa}[\![u]\!]^{(-)}(0^-)S_{\mathcal{H}_{33}} = \frac{1}{\pi}\mathcal{S}^{(s)}_{\mathcal{H}_{33}}\frac{\partial\langle p\rangle}{\partial x_1} + \langle p\rangle + \frac{\delta_3}{2\pi}\mathcal{S}^{(s)}_{\mathcal{H}_{33}}\frac{\partial[\![p]\!]}{\partial x_1} + \frac{\delta_3}{2}[\![p]\!], \quad x_1 < 0,$$

$$\langle t\rangle^{(+)} = -\frac{\mathcal{H}_{33}}{\pi\kappa}\mathcal{T}^{(c)}_{\mathcal{H}_{33}}[\![u]\!]^{(-)} - \frac{1}{\pi\kappa}[\![u]\!]^{(-)}(0^-)S_{\mathcal{H}_{33}} - \frac{1}{\pi}\mathcal{S}^{(c)}_{\mathcal{H}_{33}}\frac{\partial\langle p\rangle}{\partial x_1} - \frac{\delta_3}{2\pi}\mathcal{S}^{(c)}_{\mathcal{H}_{33}}\frac{\partial[\![p]\!]}{\partial x_1}, \quad x_1 > 0,$$

where

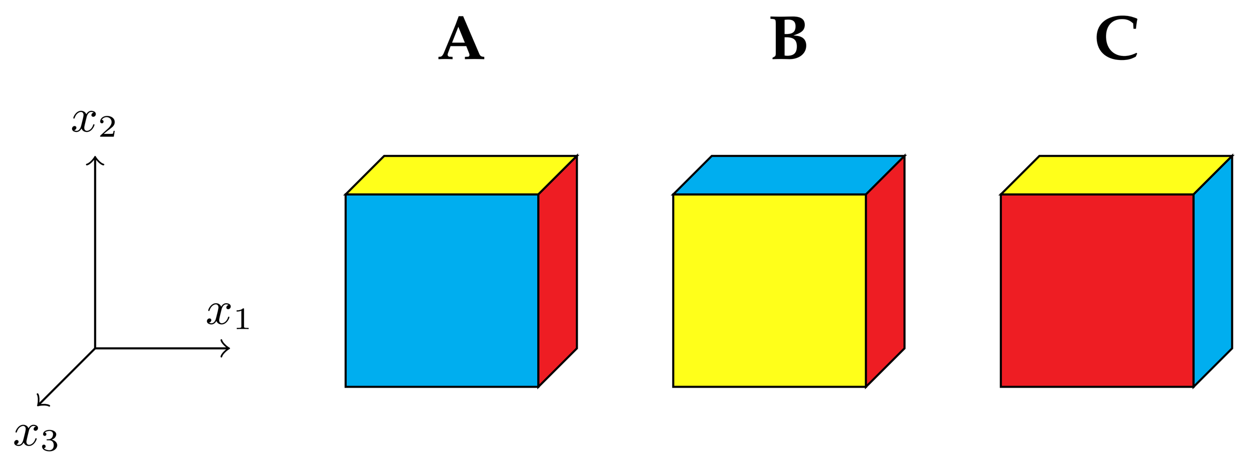
$$\mathcal{S}_{\mathcal{H}_{33}}\varphi(x_1) = (S_{\mathcal{H}_{33}} * \varphi)(x_1), \qquad \mathcal{T}_{\mathcal{H}_{33}}\varphi(x_1) = (T_{\mathcal{H}_{33}} * \varphi)(x_1), \qquad \mathcal{P}_{\pm}\varphi(x_1) = \begin{cases} \varphi(x_1) & \pm x_1 \geq 0, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$\mathcal{S}^{(s)}_{\mathcal{H}_{33}} = \mathcal{P}_-\mathcal{S}_{\mathcal{H}_{33}}\mathcal{P}_-, \qquad \mathcal{T}^{(s)}_{\mathcal{H}_{33}} = \mathcal{P}_-\mathcal{T}_{\mathcal{H}_{33}}\mathcal{P}_-, \qquad \mathcal{S}^{(c)}_{\mathcal{H}_{33}} = \mathcal{P}_+\mathcal{S}_{\mathcal{H}_{33}}\mathcal{P}_-, \qquad \mathcal{T}^{(c)}_{\mathcal{H}_{33}} = \mathcal{P}_+\mathcal{T}_{\mathcal{H}_{33}}\mathcal{P}_-.$$

It is also possible to write these identities solely in terms of $\mathcal{S}_{\mathcal{H}_{33}}$ or solely in terms of $\mathcal{T}_{\mathcal{H}_{33}}$. Then there is a choice of which sets of equations to use. Depending on the boundary conditions used certain sets of equations are more computationally efficient.

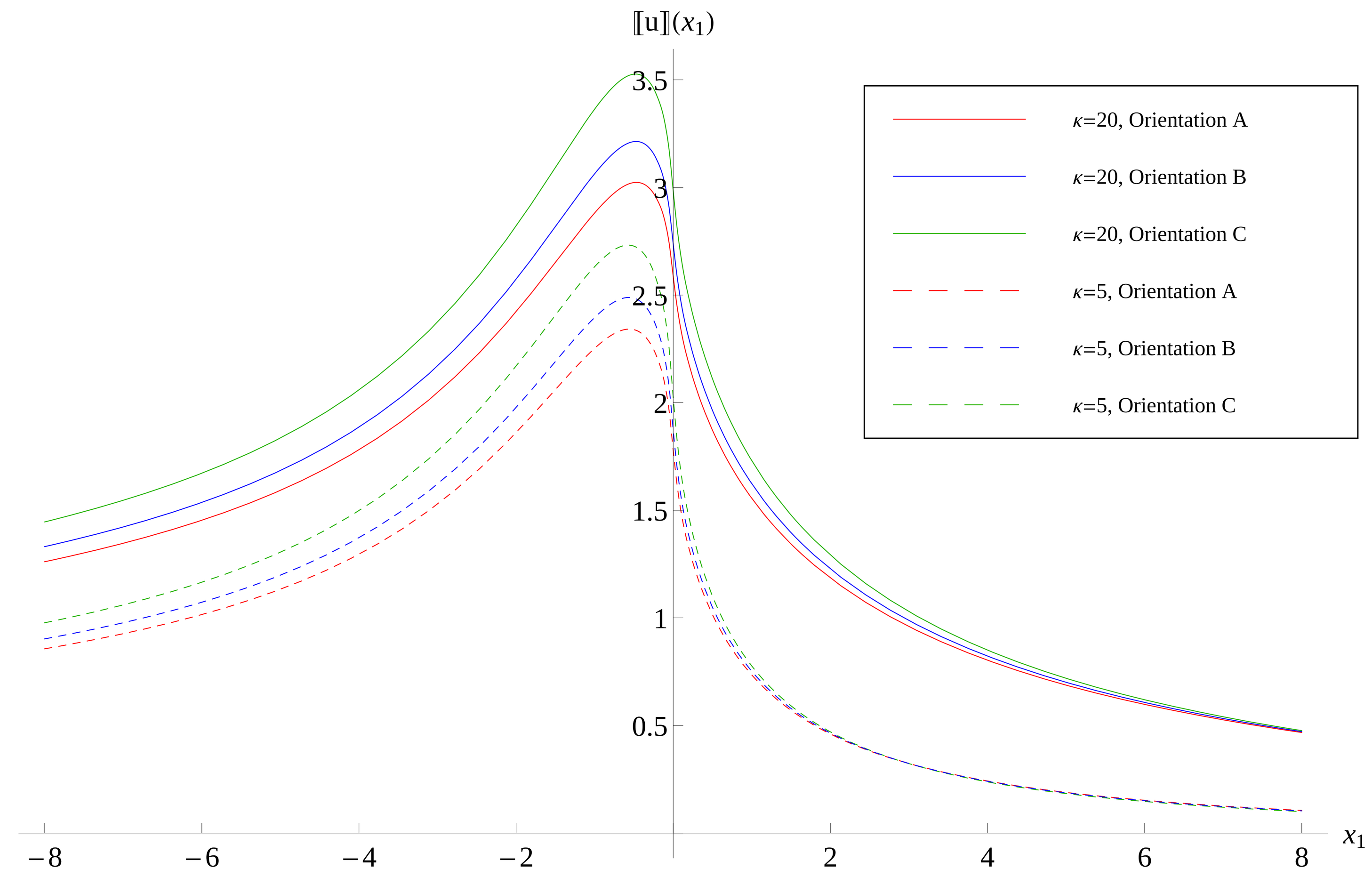
Numerical results for mode-III displacement

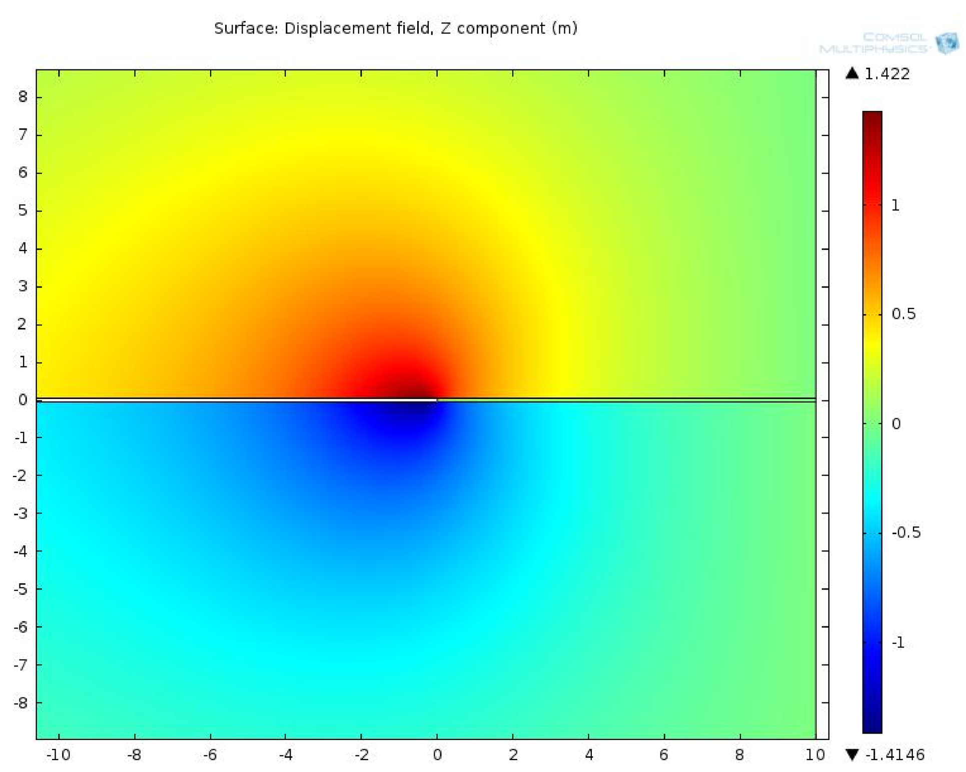


Shear moduli:	A	B	C
	$\mu_{23} = 1$	$\mu_{23} = 1$	$\mu_{23} = 1/2$
	$\mu_{13} = 2/3$	$\mu_{13} = 1/2$	$\mu_{13} = 2/3$
	$\mu_{12} = 1/2$	$\mu_{12} = 2/3$	$\mu_{12} = 1$

- Orthotropic materials used,
- Material I fixed with orientation **A**,
- Material II changed between orientations **A**, **B** and **C**,
- Two values of κ studied: 5 and 20,
- Symmetric exponential loading on the crack faces.

- Increasing κ leads to a larger jump in displacement,
- Orientation C of material II gives highest displacement jump due to lowest mode-III shear moduli. Orientation A gives lowest jump,
- For same values of κ the lines converge further along the interface for the three configurations.





- FEM simulation from COMSOL showing a colourmap of displacement,
- Imperfect interface modelled as thin layer of isotropic adhesive,
- The example shown has both materials identically orientated as **A** with $\kappa = 20$,
- Position of highest displacement (and jump) in same position as numerical results.

References

[1] Pryce, L., Morini, L., Mishuris, G. Weight function approach to a crack propagating along a bimaterial interface under arbitrary loading in an anisotropic solid. *JOMMS* **DOI: 10.2140/jomms.2013.8-8**, 479-500 (2013)
[2] Pryce, L., Morini, L., Mishuris, G. Analysis of interfacial crack propagation under asymmetric loading in anisotropic materials. *J. Phys: Conf. Series* **451**(2013)
[3] Pryce, L., Vellender, A., Zagnetko., A. Integral identities for interfacial cracks in an anisotropic bimaterial with an imperfect interface. *To be submitted*