Sludge Rheology

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Sludge Rheology - An Application

 $\longrightarrow$  $\longrightarrow$ 

Greater Manchester produces  of sludge per day

Outline of Talk

Foams vs sludges

Sludge settling theory

Sludge rheological properties

‘Diffusion’ of solids in networked sludges

Measurement of sludge rheological properties

Batch settling tests

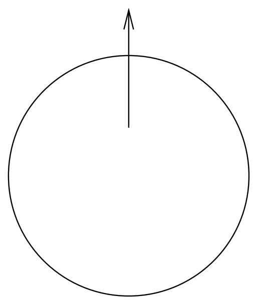
Reconstruction of solids fluxes

An explicit flux reconstruction formula

Conclusions

Context: Bubbles

Isolated bubbles rise

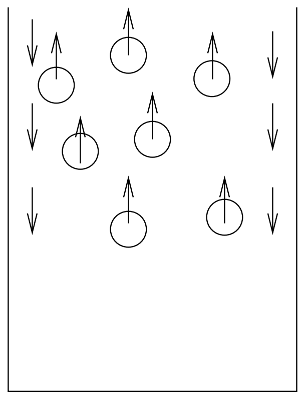


Stokes rise velocity balances:

buoyancy $\sim$ viscous drag

Context: Swarms of Bubbles

Hindered bubble rise

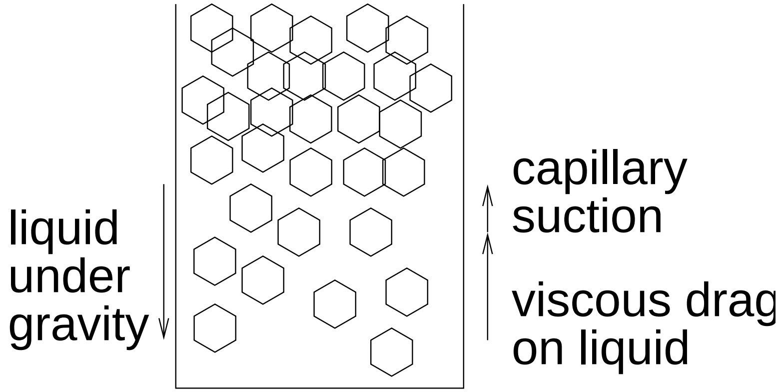


In a vessel with a closed bottom,
bubbles go up, liquid returns downward

Returning liquid holds back bubbles

Context: Densely packed bubbles (A Foam)

Liquid drains through channels between bubbles



A 'third' force is present

buoyancy + *capillary suction (wet to dry)* $\sim$ viscous drag

Capillary suction pressure P is a function of liquid fraction ε

Context: Foam Drainage Equation

Foam force balance

$$+$$

Continuity equation for liquid

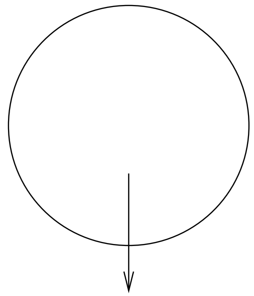
$$=$$

Foam drainage equation

(An advection-diffusion equation for liquid fraction ε)

Sludges: Upside down foams

Solid particles settle



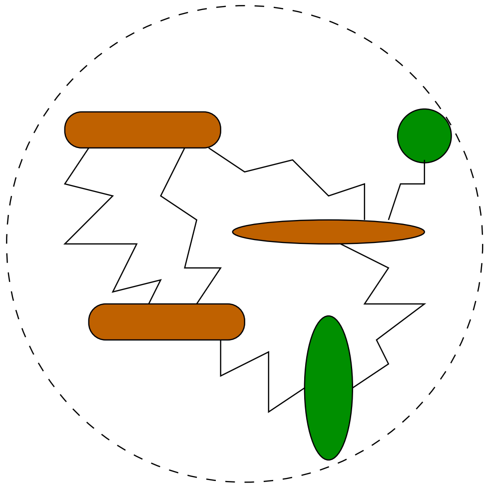
Stokes settling velocity, balances:

apparent weight $\sim$ viscous drag

Sludges: Flocculation

Particles gather together into loosely bound flocs

(bind bacteria, extracellular protein, etc.)



Flocs are the 'effective particles'

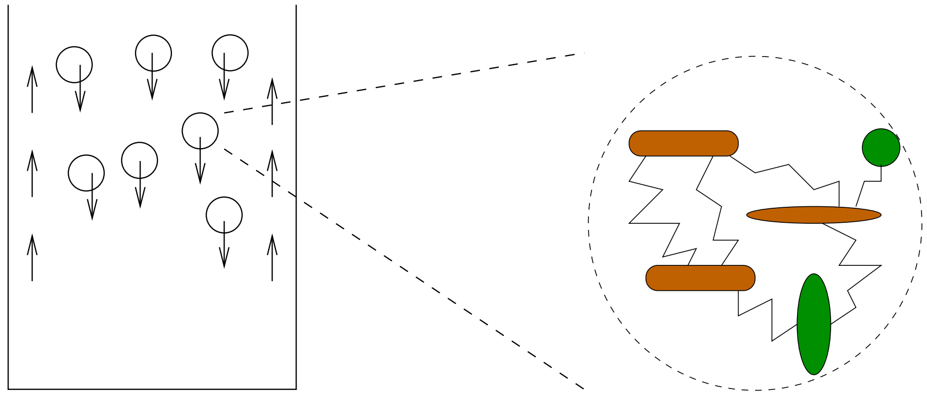
Stokes settling of flocs

Hindered settling of flocs

Due to extended nature of flocs,

effective at hindering settling

(even at relatively low solids fractions)

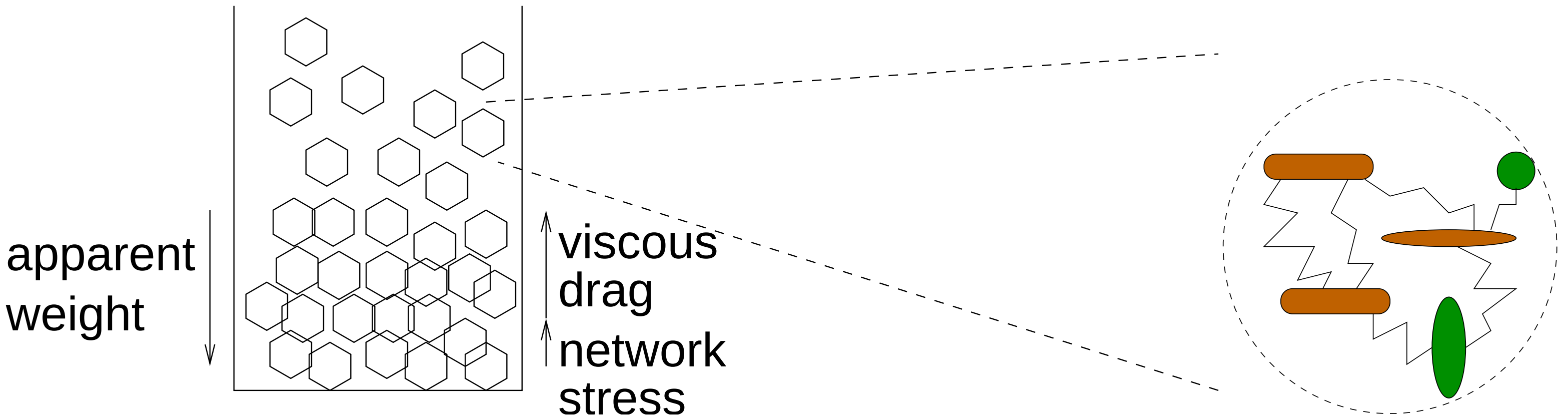


Flocs in a network

Due to extended nature of flocs,

form a weight-bearing consolidated network

(even at relatively low solids fractions)

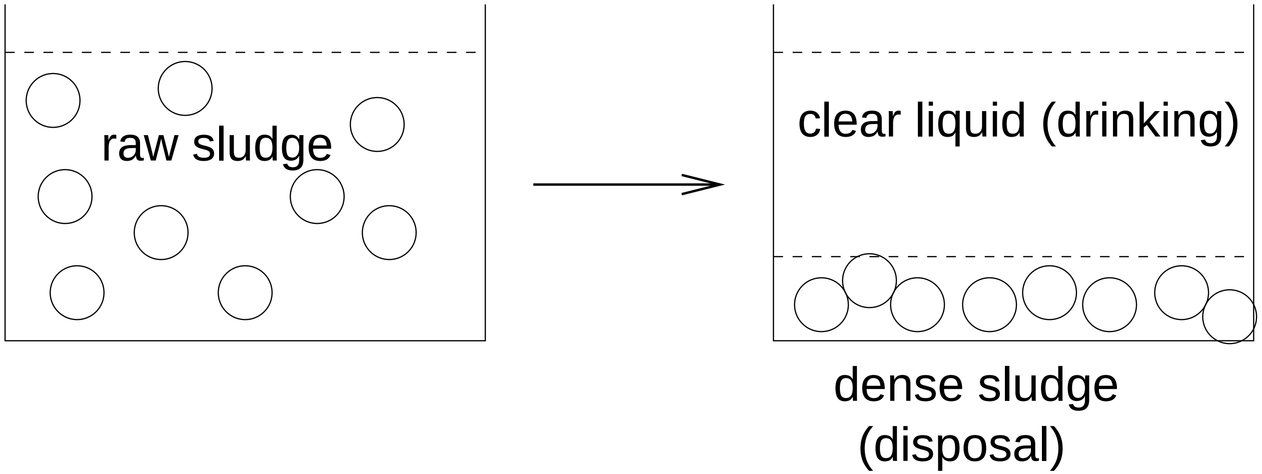


A ‘third’ force is present

apparent weight + *network stress* $\sim$ viscous drag

Dewatering

Flocs contain liquid one would like to remove



Aim: Squeeze as much liquid out of sludge,
as quickly as possible

Settling speed of flocs - Theory

Hindered settling speed u depends on solids fraction ϕ

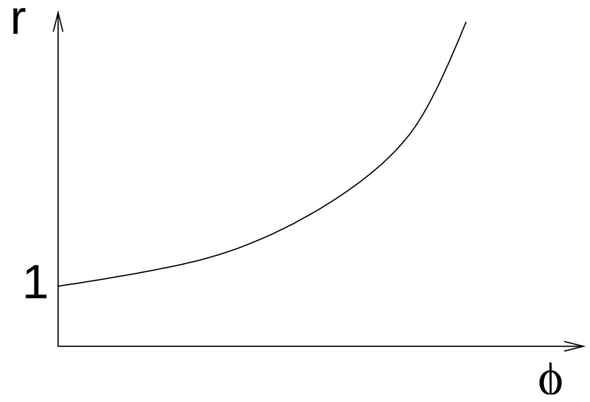
Kynch theory (1952)

$$u = \frac{u_0}{r(\phi)}$$

where u_0 is Stokes settling speed of isolated floc,

$r(\phi)$ is a hindered settling factor

(an important material property)



Settling speed in a networked suspension - Theory

Network can bear weight

Buscall and White theory (1987)

$$u = \frac{u_0}{r(\phi)}\left(1 + \frac{\mathrm{d}P/\mathrm{d}z}{\Delta\rho g\phi}\right)$$

where $\mathrm{d}P/\mathrm{d}z$ is network pressure gradient,

$\Delta\rho g\phi$ is apparent weight force of solids

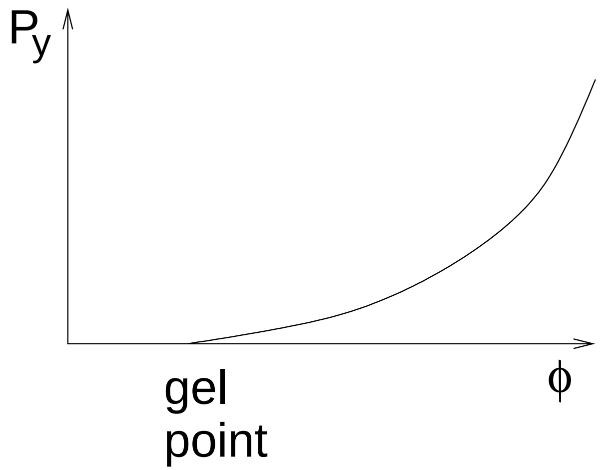
In final equilibrium state $u = 0$:

pressure gradient balances apparent weight

Network yield stress

Networked suspension is characterised by a yield stress $P_y(\phi)$

(an important material property)



Network can support *any* compressive stress P up to P_y

If P exceeds P_y, liquid is squeezed out and network dewaters

Hard vs soft materials

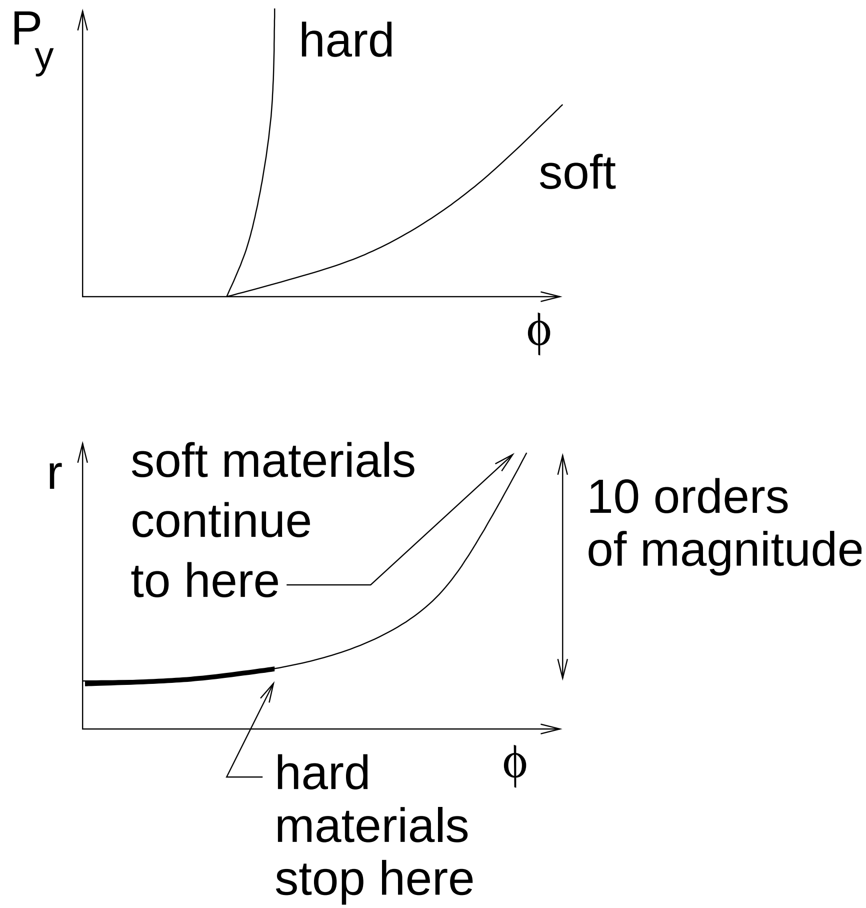
Hard materials compact to a particular solids fraction in finite time

(and stay there)

Soft materials: can always squeeze a bit more liquid out,

rate of squeezing $\downarrow$ as channels between (& thru) flocs forced shut

Hard vs soft materials: differing behaviour of $P_y(\phi)$ and $r(\phi)$



Rate of dewatering

Material derivative following a floc

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = 0, \qquad P < P_y$$

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = \kappa(\phi)(P - P_y), \qquad P > P_y$$

where $\kappa(\phi)$ is a dynamic compressibility (a material property)

Continuity equation:

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} + \phi\nabla.u = 0$$

Rate determining step for $\mathrm{D}\phi/\mathrm{D}t$

is *not* dynamic compressibility $\kappa(\phi)(P - P_y)$,

but rather the spatial variation in u

(associated with spatial changes in ϕ across the network)

Determination of network stress

When network is consolidating, $P \approx P_y(\phi)$

so that dewatering rate $\mathrm{D}\phi/\mathrm{D}t = \kappa(\phi)(P - P_y)$

is satisfied with $P - P_y \ll 1$, $\kappa \gg 1$

In general, P can be anywhere between 0 and $P_y(\phi)$,

with the network compressing plastically whenever $P = P_y(\phi)$

Settling velocity in a plastically compressing network

Yield stress gradient d$P_y(\phi)$/dz replaces pressure gradient dP/dz in Buscall and White (1987) equation

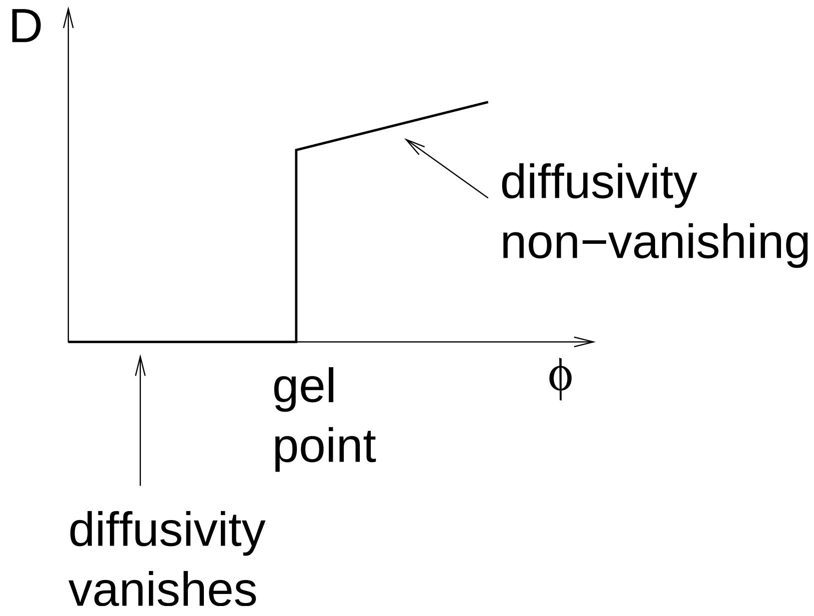
$$u = \frac{u_0}{r(\phi)}\left(1 + \frac{\mathrm{d}P_y(\phi)/\mathrm{d}z}{\Delta\rho g\phi}\right)$$
$$= \frac{u_0}{r(\phi)} + \frac{u_0\,\mathrm{d}P_y(\phi)/\mathrm{d}\phi}{\Delta\rho g\phi r(\phi)}\frac{\mathrm{d}\phi}{\mathrm{d}z}$$

$u_0\,$d$P_y(\phi)$/d$\phi/\Delta\rho g\phi r(\phi)$ behaves as a diffusion coefficient $D(\phi)$

An advection-diffusion equation results for solids fraction ϕ

(analogous to the foam drainage equation for liquid fraction ε)

Diffusion, but not as we know it!



In some parts of the sludge $P_y = 0 \longrightarrow \mathrm{d}P_y(\phi)/\mathrm{d}\phi \longrightarrow D(\phi) = 0$

(in that case, pure advection, rather than advection-diffusion)

Sludge rheology from a modeller's viewpoint:

Deducing sludge material properties $P_y(\phi)$, $r(\phi)$ and $D(\phi)$
from experimental sludge characterisation tests
(i.e. solving inverse problems)

Once sludge material properties are known,
solving mixed advection/advection-diffusion equations
to predict performance of various dewatering equipment
(settlers, thickeners, filter presses, centrifuges)

Selecting and designing the best dewatering equipment
for a sludge with given material properties

Measurement of Sludge Rheological Properties

Experimentally measure $P_y(\phi)$ & $r(\phi)$ on the laboratory scale

$\downarrow$

Robustly design dewatering equipment on the engineering scale

Measurement of $P_y(\phi)$ (Green et al. 1996; Usher et al. 2001)

$P_y(\phi)$ is a steady state property

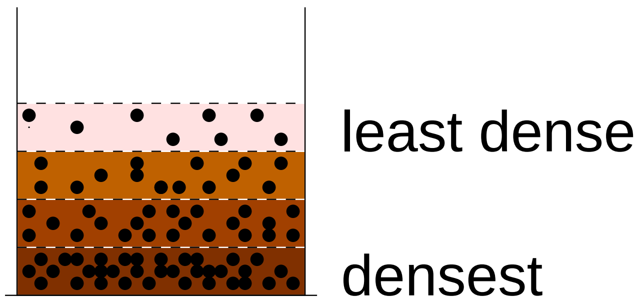
Settle sludge to steady state

At steady state, vanishing settling velocity

$$u \equiv 0 \quad \longrightarrow \quad \left(1 + \frac{\mathrm{d}P_y(\phi)/\mathrm{d}z}{\Delta\rho g\phi}\right) = 0$$

Obtain $P_y(\phi)$ via a scrape test,
measuring ϕ layer by layer

One experiment furnishes
P_y values for many different ϕ



Measurement of hindered settling $r(\phi)$ and/or settling flux

$r(\phi)$ is an inherently dynamic property

(associated with differential motion between liquid and gas)

Simplest to determine for an unnetworked suspension

(at relatively low ϕ)

$$u = \frac{u_0}{r(\phi)}$$

Settling flux:

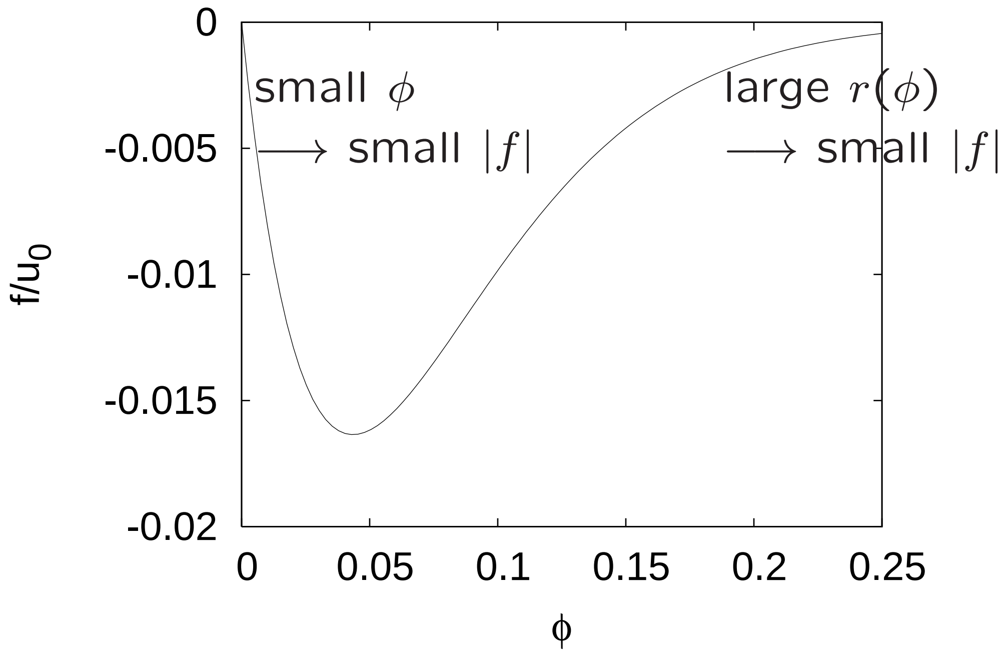
Engineers are less concerned with settling speed $u = u_0/r(\phi)$

and more concerned with settling flux

$$f(\phi) = \phi u = \frac{\phi u_0}{r(\phi)}$$

Knowing $f(\phi)$ is equivalent to knowing $r(\phi)$

Typical settling flux curve $f \equiv \phi u_0 / r(\phi)$ vs ϕ

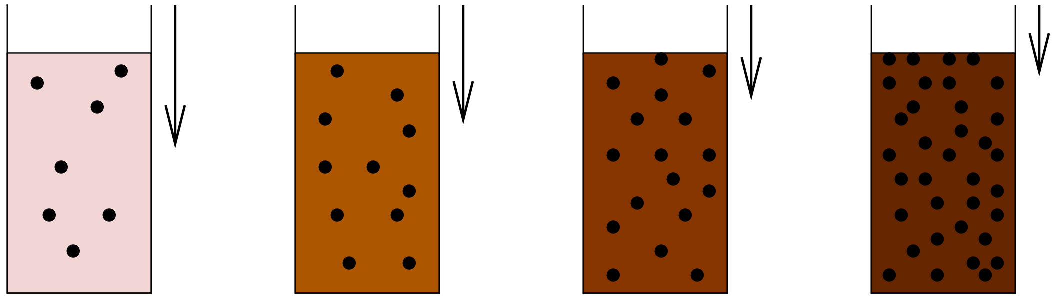


Optimal (i.e. maximal) settling flux $|f|$ at a particular value of ϕ

Batch settling test:

Make multiple suspensions at many different ϕ values

Measure initial settling speed



least dense (fastest settling)

densest (slowest settling)

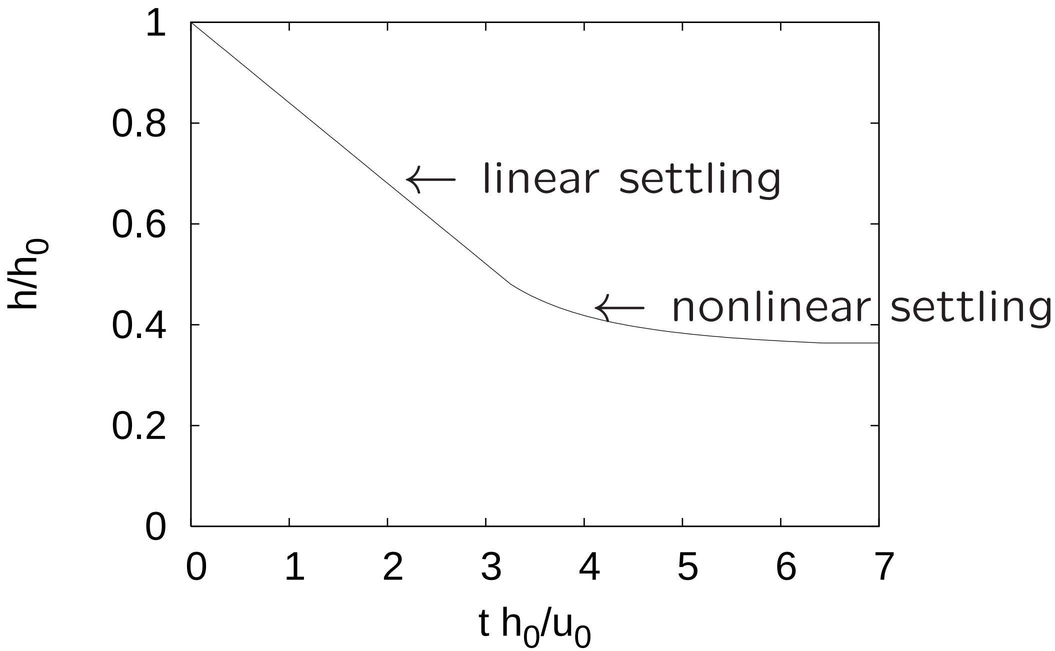
Experiment furnishes r value for only one single value of ϕ

Examine batch settling data closely:

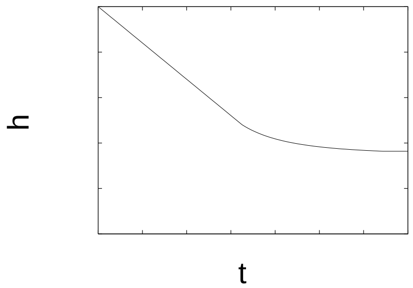
In a batch settling test, settling speed is initially constant

but subsequently changes over time

Settling height switches from linear to nonlinear with respect to time

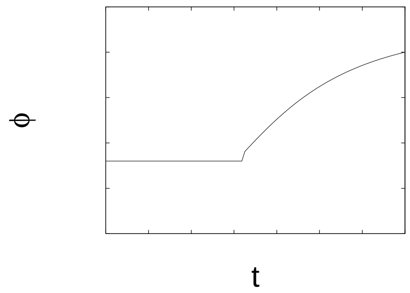


Changes in batch settling speed over time



Settling speed changes because. . .

. . . solids fraction ϕ
at the suspension surface changes

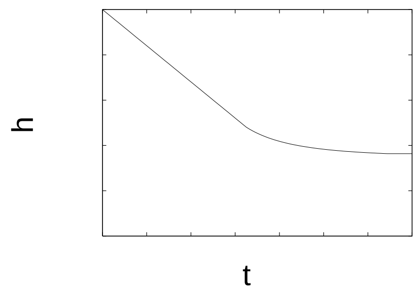


In principle, single experiment could furnish r
for many values of ϕ (Lester et al. 2005)

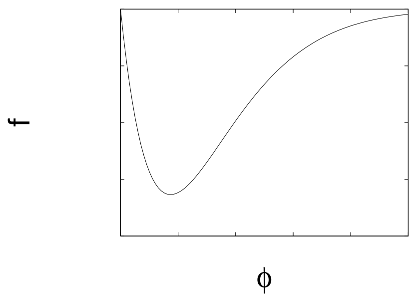
Use batch settling data intelligently
to minimise experimentation required

Batch settling: The Challenge

Measure settling height h vs time t



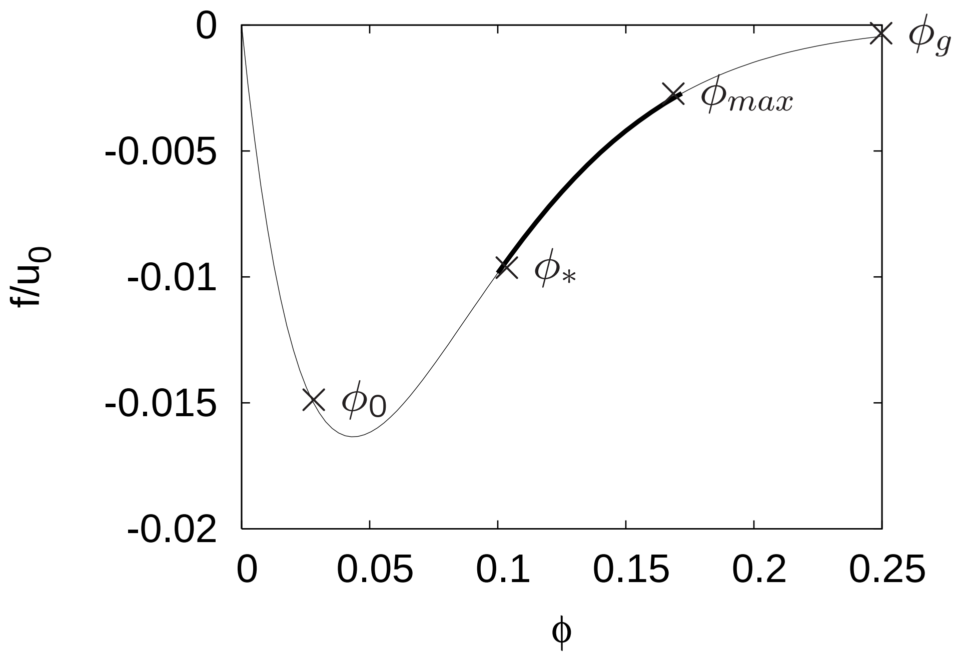
$\downarrow$



Reconstruct settling height f
vs solids fraction ϕ
for a range of solids fractions

Which range?

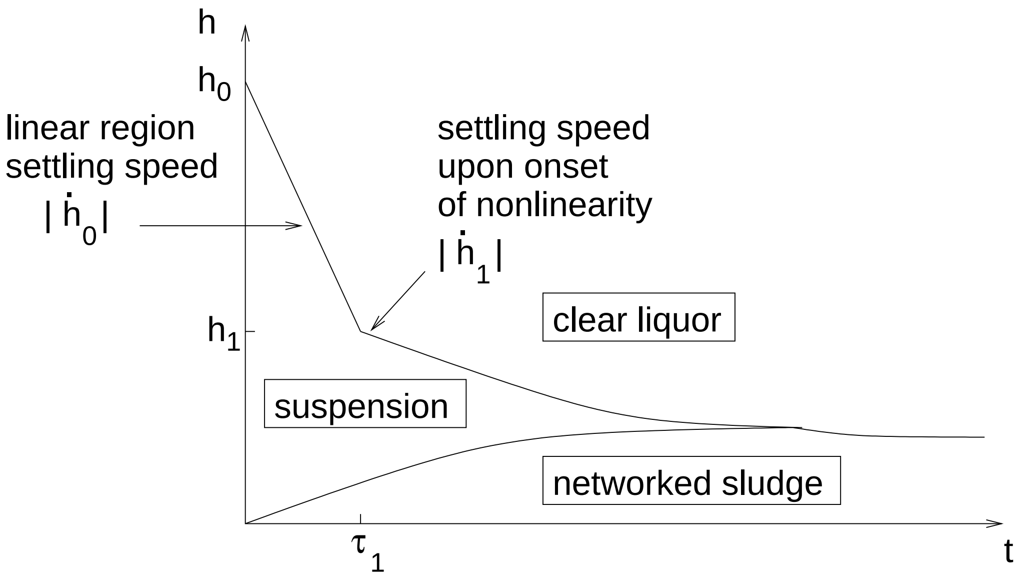
Portion of $f(\phi)$ curve to be reconstructed



Typically reconstruct from some ϕ_*
(greater than initial solids fraction ϕ_0)
up to ϕ_{max}
(less than network gel point ϕ_g)

Lower limit of reconstruction range ϕ_*:

Typically a jump in solids fraction from ϕ_0 to a higher value ϕ_* is associated with a jump in settling speed from $|\dot{h}_0|$ to $|\dot{h}_1|$; this occurs at time τ_1 and suspension height h_1



$$\phi_* = \frac{(h_1 + |\dot{h}_0|\tau_1)}{(h_1 + |\dot{h}_1|\tau_1)}\phi_0$$

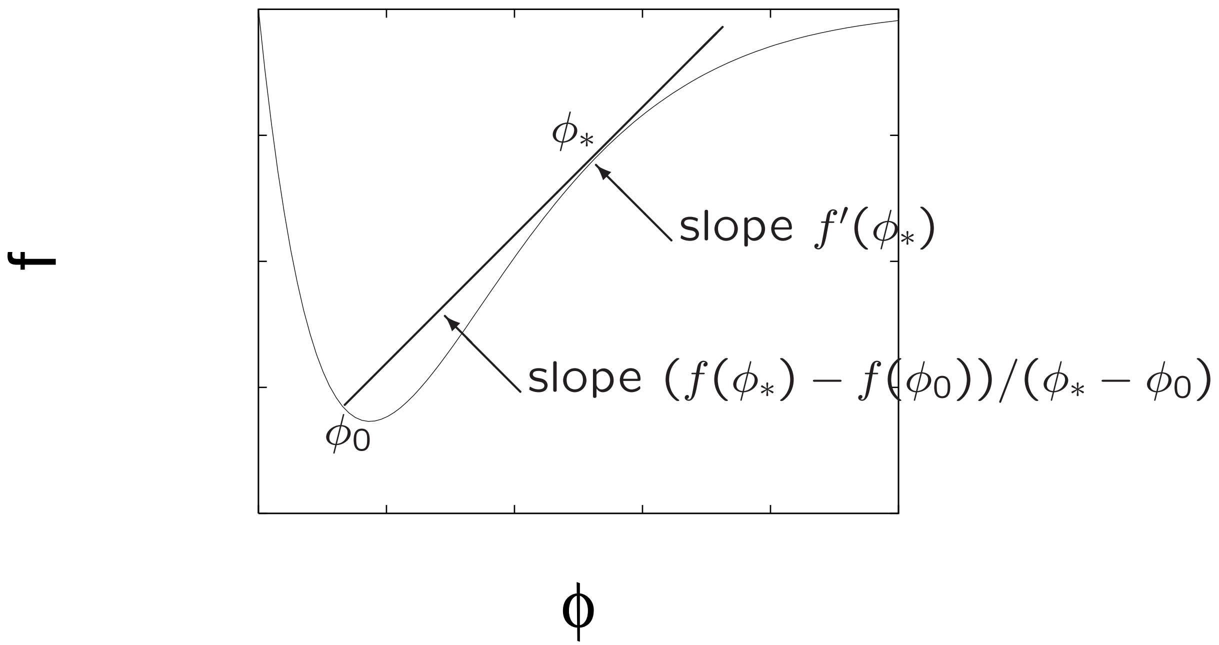
Determining jump in solids fraction - Theory of kinematic waves:

Velocity matching based on properties of settling flux

Group velocity $f'(\phi_*)$ associated with solids fraction ϕ_*

matches Rankine-Hugoniot velocity $(f(\phi_*) - f(\phi_0))/(\phi_* - \phi_0)$

of discontinuity between ϕ_0 and ϕ_*



Upper limit of reconstruction range ϕ_{max}:

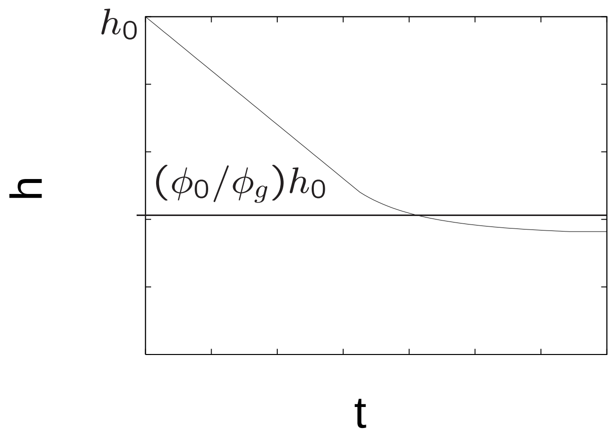
Typically less than the suspension network gel point ϕ_g

Highest solids fraction for which settling curve remains unaffected by the presence of networked sludge at the base of the suspension

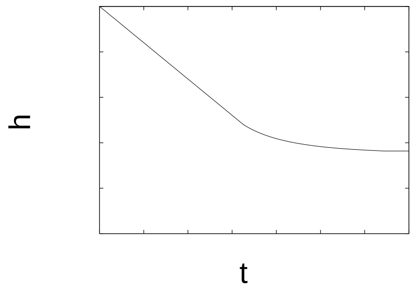
ϕ_{max} corresponds to a cut-off suspension settling height (Grassia et al. 2008)

$$h_{cut\text{-}off} = \frac{\phi_0}{\phi_g} h_0$$

$h_{cut\text{-}off}$ is sensitive to ϕ_g, but not to the details of $P_y(\phi)$ for $\phi > \phi_g$



Batch settling: Measurements & Unknowns



Measure settling height h vs time t

Changes in velocity $\dot{h}$ reflect

changes in $u \equiv f/\phi$

over a known range of solids fraction ϕ

as achieved at the suspension surface

However instantaneous values ϕ

at suspension surface

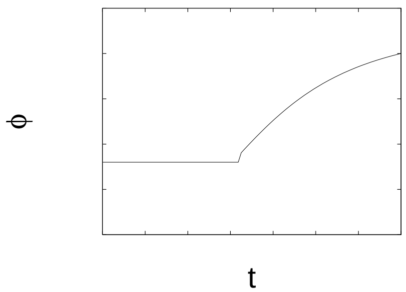
are a priori unknown

(and, being dynamic,

are difficult to measure)

$\longrightarrow$ reconstruction must solve for ϕ

in addition to settling flux f



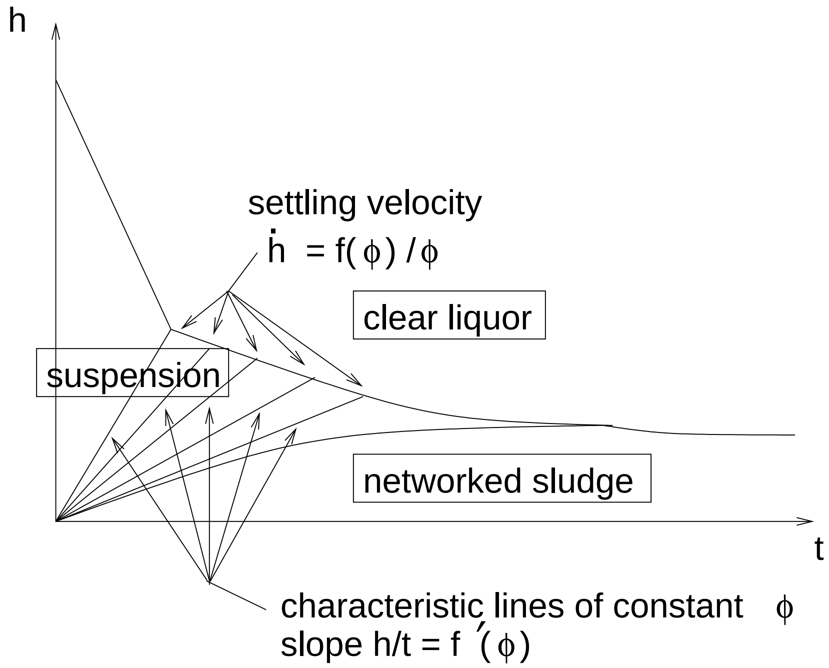
Eqns to solve governing batch settling tests (Lester et al. 2005)

Instantaneous settling velocity

$$\dot{h} = u(\phi) = \frac{f(\phi)}{\phi}$$

Characteristic lines propagate up from the bottom of the sludge with a slope given by the group velocity $f'(\phi)$

$$\frac{h}{t} = f'(\phi)$$



Solutions for ϕ and f (Diehl 2007)

$$\phi = \frac{\phi_0 h_0}{h - \dot{h}t}, \quad f = \phi u = \frac{\phi_0 h_0 \dot{h}}{h - \dot{h}t}$$

where ϕ_0 = initial solids fraction, h_0 = initial suspension height

Easy to check (as required)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{h}{t}\frac{\mathrm{d}\phi}{\mathrm{d}t} \quad \longrightarrow \quad \frac{h}{t} = f'(\phi)$$

A parametric solution for ϕ and f in terms of t

Sensitive to experimental noise (especially in $\dot{h}$)

An ‘exact’ reconstruction procedure

to within the limitations of the experimental noise

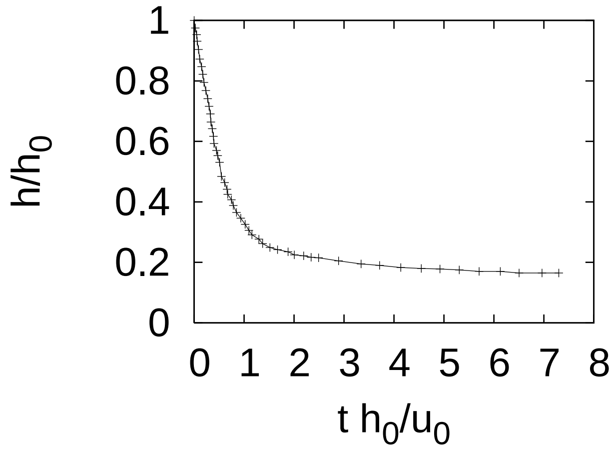
Examples of experimental noise

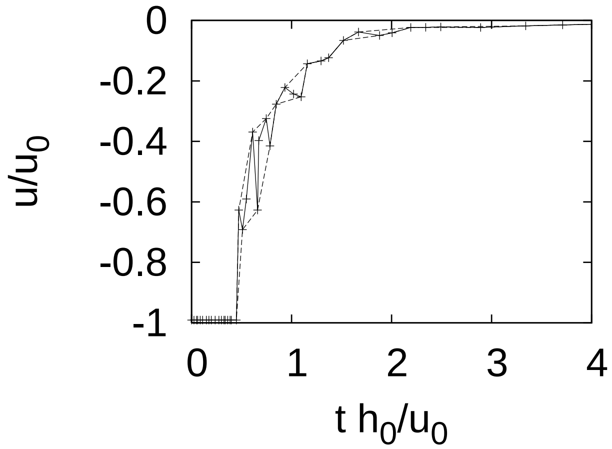
Settling experiments on a calcium carbonate suspension

(Data of Gladman et al. 2006)

Small amount of noise in h vs t

Noise in u vs t is considerable





Reducing sensitivity to experimental noise (Grassia et al. 2008)

Power law fit to the settling height & velocity functions

$$h = h_1 - \frac{\tau_1|\dot{h}_1|}{\beta} + \frac{\tau_1|\dot{h}_1|}{\beta}\frac{\tau_1^{\beta}}{t^{\beta}}, \quad u = -|\dot{h}_1|\frac{\tau_1^{(\beta+1)}}{t^{(\beta+1)}}$$

where τ_1 = time at which $\dot{h}$ begins to vary,

h_1 = settling height at which $\dot{h}$ begins to vary,

$\dot{h}_1$ = corresponding settling velocity, β = power law fitting exponent

Reduced sensitivity to noise

(since fitting eliminates fluctuations of individual $\dot{h}$ values)

Settling flux reconstruction based on power law fits

Explicit formula $f = f(\phi)$

$$f \;=\; -\phi|\dot{h}_1|\left(\frac{\beta}{\beta+1}\right)^{(\beta+1)/\beta}\left(\frac{h_0}{\tau_1|\dot{h}_1|}\frac{\phi_0}{\phi}+\frac{1}{\beta}-\frac{h_1}{\tau_1|\dot{h}_1|}\right)^{(\beta+1)/\beta}$$

An approximate reconstruction formula

to the extent that

settling height data h vs t

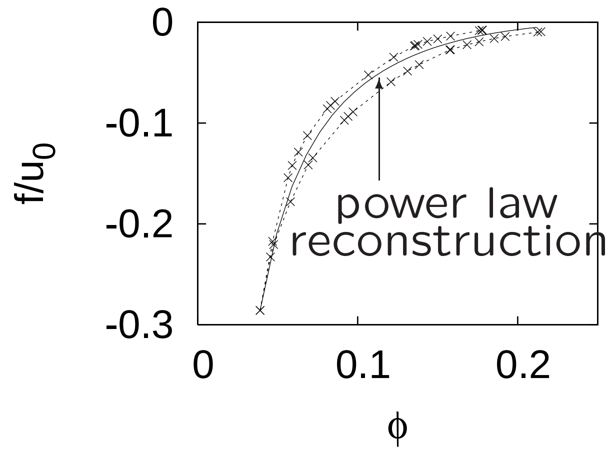
are well fit via a power law

Flux reconstructions from fits to experimental settling data

(Again data of Gladman et al. 2006 - with $\phi_0 = 0.04$)

Power law reconstruction technique within envelope of experimental noise

No significant loss of accuracy & explicit formula for $f(\phi)$ is available



Conclusions - Sludge Rheology

Sludges are 'upside-down foams'

Sludges are characterised by two rheological functions
hindered settling factor and compressive yield stress
(both of which depend on solids fraction)

Knowledge of the rheological functions permits
confident and robust design of engineering equipment

Sludge dewatering rate governed by spatial variations in
solids fraction (inducing spatial variations in rheological functions),
rather than by excess of imposed pressure
over and above the local yield stress
$\longrightarrow$ Networked sludge superimposes a diffusive solids flux
onto a convective buoyant flux

Conclusions - Measurement of Sludge Rheological Properties

Compressive yield stress obtained from experimental scrape test

Hindered settling function obtained from batch settling test

A single settling test provides hindered settling data
over a range of solids fractions

Relevant range of solids fractions
for reconstruction of hindered settling factor can be found
independently of the compressive yield stress

Approximate reconstruction formulae
(e.g. based on power law fits to batch settling height data),
provide explicit functional forms of the settling flux function that are
well within the noise of experimental measurements